Price, Strategy, and Market Friction

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About Me

- **Background:**
  - PhD: Dept. of Math, National University of Singapore;
  - Postdoc: Dept. of Math, ETH Zürich;
  - Assistant Professor: SEEM CUHK since 2019 Fall.
- **Research Area:** Market Frictions, Portfolio Selection, FinTech

Market Friction Modeling: An Overview

Price

Frictions

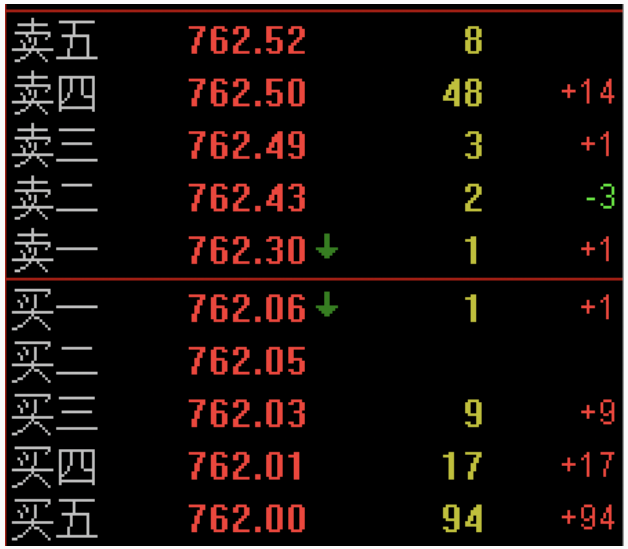
Transaction Costs
Price Impact
Capital Gains Tax
Market Closure

Strategy

Portfolio Allocation
Hedging
Market Making

Market Frictions

- **Transaction Costs:** costs per transaction (fixed or proportional to the transaction amount);
  - Charged by broker; Bid-ask spread.
- **Price Impact:** purchasing or selling moves the price against the trader;
  - Inversely related to liquidity.
- **Capital Gains Tax:** paid for realizing capital gains, but rebated for realizing capital losses;
  - Rate can be as high as 40%.



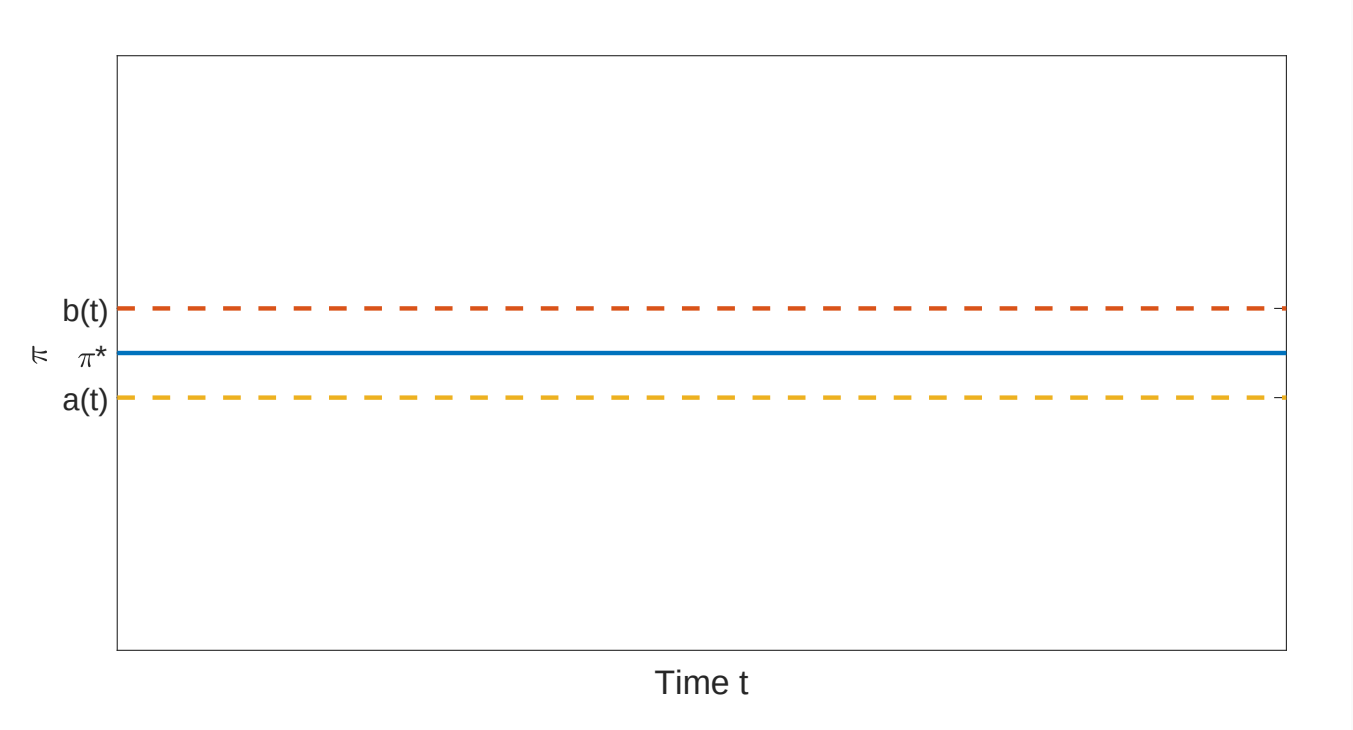
Merton Model

Consider the classic finite-horizon Merton model

- Stock price
$$dS_t = \mu S_t dt + \sigma S_t dB_t.$$
- Portfolio value W_t, for which π_t is the fraction of W_t in stock
$$dW_t = (1-\pi_t) r W_t dt + \pi_t(\mu W_t dt + \sigma W_t dB_t).$$
- Objective:
$$\sup_{\pi} \mathbb{E}[\mathcal{U}(W_T)], \;\; \mathcal{U}(x) = \frac{x^{1-\gamma}}{1-\gamma}, \; \gamma \neq 1.$$
- Solution:
$$\pi_t^* = \frac{\mu - r}{\gamma \sigma^2}.$$
- **Economic Implication:** Optimal risk exposure via optimal return-risk trade-off.

Example: Classic Proportional Transaction Cost Model

- Optimal strategy: there exist $a(t) < b(t)$, such that:
  - $\pi_t < a(t)$: Buy;
  - $\pi_t > b(t)$: Sell;
  - $a(t) \leq \pi_t \leq b(t)$: No transaction.
- **Economic Implication:** Trade-off between optimal exposure and cost.



Market Friction Modeling: An Overview

LETF Rebalancing (MS, Forthcoming)

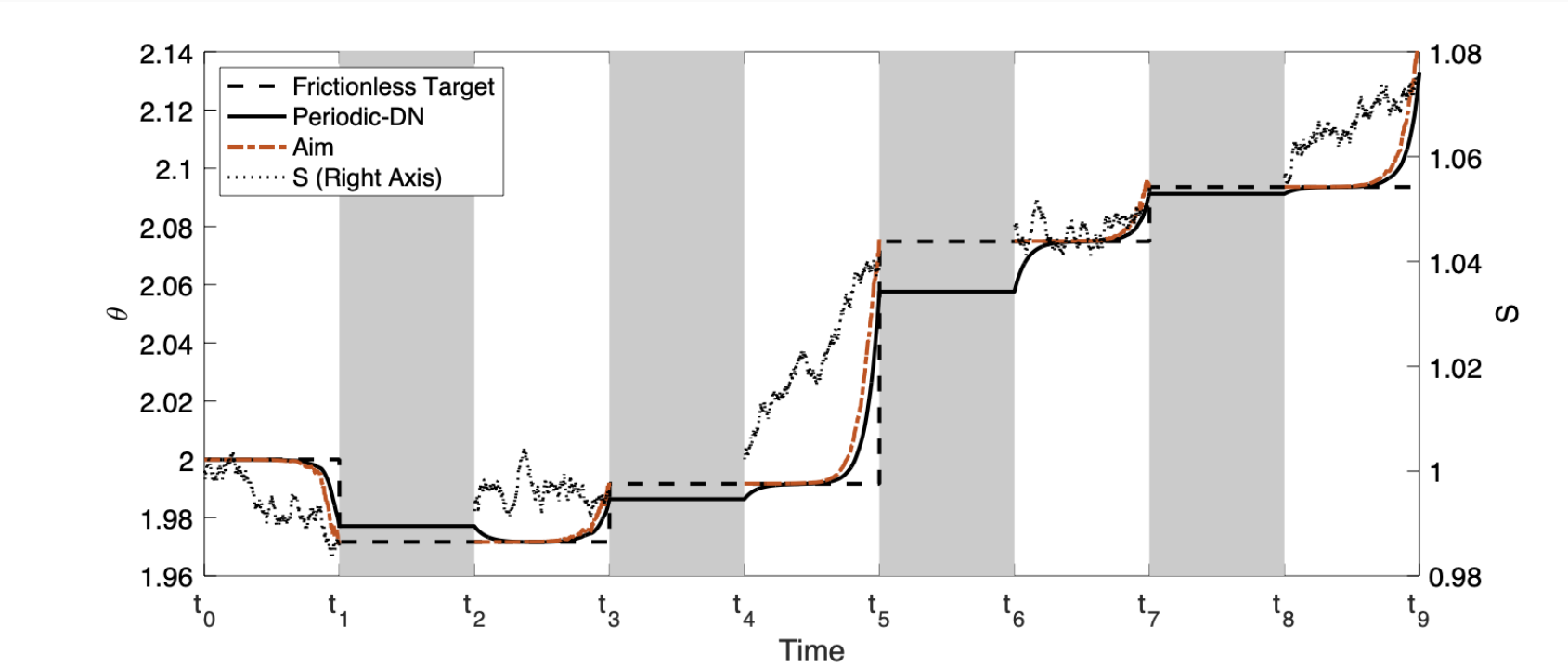
Min Dai, Steven Kou, Mete Soner and Chen Yang, Leveraged ETFs with Market Closure and Frictions, **Management Science**, forthcoming.

- Leveraged ETFs: track β times the index **daily** return;
- Continuous control, discrete monitoring;
- State: $(t, S, X, \theta; \bar{S}, \bar{X})$
- Goal:

$$\min_{\varphi} \sum_{i=0}^{\infty} e^{-\rho t_i} E\left[\frac{1}{2} X_{t_i}^2 \left(R_{t_{i+1}}^X - \beta R_{t_{i+1}}^S\right)^2 + \int_{t_i}^{t_{i+1}} \frac{\Lambda}{2} S_u^2 \varphi_u^2 du\right]$$

LETF Rebalancing (MS, Forthcoming)

- **Economic Implication:**
  - Trade-off between (1) current optimal exposure; (2) future optimal exposure; (3) trading speed to minimize cost.
  - The model can match the level of empirical slippage and explain the weekend effect.

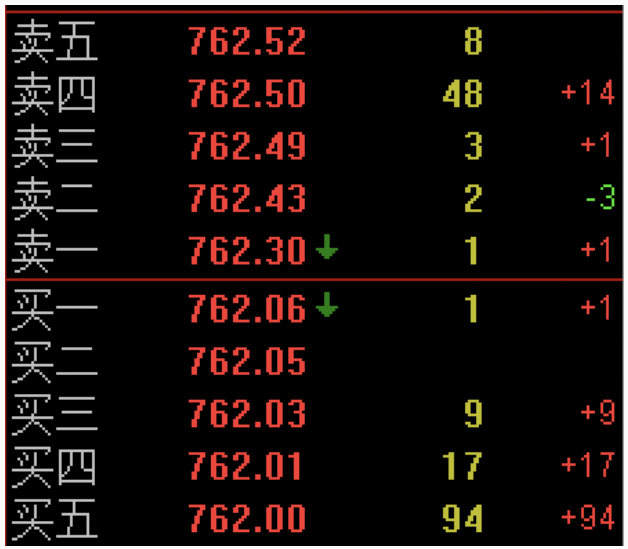


Market Friction Modeling: An Overview

Optimal Trading on LOB (Working Paper)

Nan Chen, Min Dai, Qiheng Ding and Chen Yang, Optimal Investment under Block-Shaped Order Books, **Working Paper**.

- Trading increases the spread, but the spread will reduce over time due to resilience.
- Therefore, timing the trade can lead to lower trading cost.
- **Economic Implication:** Trade-off between (a) current optimal exposure; (b) future optimal exposure; (c) trade timing to minimize cost



Market Friction Modeling: An Overview

Capital Gains Tax Model (RFS 2015)

Min Dai, Hong Liu, Chen Yang, and Yifei Zhong, Optimal Tax-timing with Asymmetric Long-term/short-term Capital Gains Tax, **The Review of Financial Studies**, 28.9:2687-2721, 2015.

- Denote K_t as the purchasing cost. Selling $dM_t \in (0,1]$ fraction of stocks incurs capital gains tax
$$\alpha(Y_t - K_t)dM_t.$$
- Existing theoretical model: investors should realize losses immediately to get tax rebate.
- Empirical puzzle: many investors hold onto short-term and even long-term losses.
- **Economic Implication:** our model explains the empirical observation via asymmetric long/short-term rates and realistic tax code features.

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Liquidity Premia (MOR, Forthcoming)

Johannes Muhle-Karbe, Xiaofei Shi, and Chen Yang, An Equilibrium Model for the Cross-Section of Liquidity Premia, **Mathematics of Operations Research**, forthcoming.

- Price process $dS_t = \mu_t dt + \sigma_t dB_t$.
- $(\mu, \sigma) \rightarrow$ investor's strategy φ^i (maximizing return, minimizing risk and trading cost).
- Market clearing: $\sum \varphi^i = s$.
- **Economic Implication:** "Liquidity CAPM" involving the impact of market risk, market frictions, heterogeneity in investor risk preference, and individual trading motives.

$$\mu \sim \bar{\gamma}\alpha\alpha^\top s + \lambda^{1/2} \frac{\gamma^2 - \gamma^1}{\sqrt{2(\gamma^1 + \gamma^2)}} \left[\bar{\gamma}\bar{\Lambda}A^{-1}\alpha(\xi + \xi^\top)\alpha^\top s + A\dot{\bar{\varphi}}\right].$$

Market Friction Modeling: An Overview

MM in High-Frequency Trading (SIFIN 2020)

Sebastian Herrmann, Johannes Muhle-Karbe, Dapeng Shang, and Chen Yang, Inventory Management for High-frequency Trading with Imperfect Competition, **SIAM Journal on Financial Mathematics**, 11(1):1-26, 2020.

- High-Frequency traders
  - have inside information about fundamental price S;
  - maximize return and minimize inventory;
  - Nash competition under **inventory aversion**.
- Market maker
  - no inside information;
  - set execution price P.
- In equilibrium: $P_n - S_{n-1} = \lambda \Delta L_n + \mu M_{n-1}$
- **Economic Implication:**
  - Competition and risk aversion cause HFTs to overexploit the inside info, which improves market liquidity;
  - penalizing HFT by increasing transaction taxes in fact decreases market liquidity.

Reference

- Min Dai, Hong Liu, Chen Yang, and Yifei Zhong, Optimal Tax-timing with Asymmetric Long-term/short-term Capital Gains Tax, *The Review of Financial Studies*, 28.9:2687-2721, 2015.
- Min Dai, Steven Kou, Mete Soner and Chen Yang, Leveraged ETFs with Market Closure and Frictions, *Management Science*, forthcoming
- Johannes Muhle-Karbe, Xiaofei Shi, and Chen Yang, An Equilibrium Model for the Cross-Section of Liquidity Premia, *Mathematics of Operations Research*, forthcoming
- Sebastian Herrmann, Johannes Muhle-Karbe, Dapeng Shang, and Chen Yang, Inventory Management for High-frequency Trading with Imperfect Competition, *SIAM Journal on Financial Mathematics*, 11(1):1-26, 2020.
- Nan Chen, Min Dai, Qiheng Ding and Chen Yang, Optimal Investment under Block-Shaped Order Books, working paper.
- Yizhou Cao, Min Dai, Steven Kou, Lewei Li, and Chen Yang, Designing Stable Coins, submitted.
- Min Dai, Steven Kou, and Chen Yang, A Stochastic Representation for Nonlocal Parabolic PDEs with Applications, *Mathematics of Operations Research*, forthcoming.

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